Vibration Analysis of a Rolling Tire-wheel System

The tire-wheel system is an important component of a passenger car. Continental, Polytec and Daimler analyze the three-dimensional vibrations of this complex system attached to the front axle of an executive-class car. A mathematical model of the rolling tire-wheel is derived using the FE method and coupled to a model of the vehicle. Measurements with a 3-D scanning laser Doppler vibrometer show the validity of the models for the NVH performance of the complete vehicle.

COMPLEX – MORE THAN 20 COMPONENTS FOR A TIRE

The primary function of a tire-wheel system is to support the vehicle load, and to transmit the forces to the road required to accelerate, brake and turn corners. Conversely, it is also responsible for the reduction of vibrations and noise, experienced by the occupants due to irregularities of the road surface. With the reduction of combustion engine noise, the advent of alternative drive systems, and weight reduction of components, this secondary function is only growing in importance.

In addition to the increasing demands on the Noise, Vibration, and Harshness (NVH) performance of the tire, there is a strong demand for the reduction of development costs and time, and therefore a need for accurate and efficient numerical methods to aid the development process.

The understanding and mathematical modeling of the tire-wheel system are challenging for several reasons. Firstly,

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although the tire, wheel, and air cavity in the tire might be considered as two solid structures and an enclosed fluid, the tire alone consists of over 20 components. Each component comprises of one or more non-linear materials, whose properties depend on temperature, strain rate, strain amplitude and strain history. Secondly, excitation of the tire occurs at the contact with the road and is therefore a function of both the road profile and the non-linear dynamic response of the tire at this interface.

For small amplitudes and frequencies between 20 and 400 Hz, a modal model can be used to efficiently describe the response of the tire and wheel. Here, it will be shown that the most general form of modal model is required for the rolling tire-wheel system due to the complicated distribution of damping in the tire, gyroscopic forces generated by rolling, and coupling between the tire-wheel structure and the air cavity within the tire.

The experimental setup is designed for the purpose of applying a controlled kinematic excitation to a single wheel of the vehicle. Using a 3-D scanning laser Doppler measurement system and a judiciously placed mirror, it is possible to measure most of the visible surface of the rotating tire and wheel. The accurate measurement of amplitudes is needed for both the deeper understanding of the dynamic properties of the system, and for the validation of a mathematical model with all the complexities and assumptions.

MATHEMATICAL MODEL

A FE model leads to a description of the physical properties in terms of a structural mass matrix M_s, structural complex stiffness matrix $\tilde{K}_s$, fluid mass matrix M_f and fluid complex stiffness matrix $\tilde{K}_f$. Due to the coupling between the air cavity and the structure (described by the matrix H_{sf}), the system matrices are non-symmetric [1]. After the non-linear static loading of the tire-wheel system and the application of rolling using the Arbitrary Lagrangian-Eulerian (ALE) method, the equation of motion can be expressed as in Eq. 1:

Eq. 1

$$\begin{bmatrix} M_s & 0 \\ \rho c^2 H_{sf}^T & M_f \end{bmatrix}\begin{Bmatrix}\ddot{x}(t)\\ \ddot{p}(t)\end{Bmatrix} + \begin{bmatrix} G_s & 0\\ 0 & G_f\end{bmatrix}\begin{Bmatrix}\dot{x}(t)\\ \dot{p}(t)\end{Bmatrix} + \begin{bmatrix}\tilde{K}_s & -H_{sf}\\ 0 & \tilde{K}_f\end{bmatrix}\begin{Bmatrix}x(t)\\ p(t)\end{Bmatrix} = \begin{Bmatrix} f_s(t)\\ f_f(t)\end{Bmatrix}$$

Here, ρ is the density of air, c is the speed of sound, G_s is the sturcture's skew-symmetric gyroscopic matrix, and G_f is the fluid's skew-symmetric gyroscopic matrix. The time-varying quantities are displacement x, acoustic pressure p, and force f. To get to the standard eigenvalue form [2] the $N \times N$ second-order differential equations in Eq. 1 need to be converted into the $2N \times 2N$ first-order differential equations according to Eq. 2 (a) and Eq. 2 (b):

Eq. 2 (a)

$$\begin{bmatrix} G_s & M_s & 0 & 0\\ M_s & 0 & 0 & 0\\ 0 & \rho c^2 H_{sf}^T & G_f & M_f\\ 0 & 0 & M_f & 0\end{bmatrix}\{\dot{u}(t)\} + \begin{bmatrix}\tilde{K}_s & 0 & -H_{sf} & 0\\ 0 & -M_s & 0 & 0\\ 0 & 0 & \tilde{K}_f & 0\\ 0 & 0 & 0 & -M_f\end{bmatrix}\{u(t)\} = \{f(t)\}$$

Eq. 2 (b)

$$\{u(t)\} = \begin{Bmatrix} x(t)\\ \dot{x}(t)\\ p(t)\\ \dot{p}(t)\end{Bmatrix}$$

Due to the non-symmetric matrices in Eq. 2, the eigensolution will contain the single eigenvalue matrix $[s_r]$ and two sets of complex eigenvectors $[\Theta_{LH}]$ and $[\Theta_{RH}]$. The right-hand eigenvector (RH) describes the mode shapes and the left-hand eigenvector (LH) describes the excitation shapes. The complex modes not only have amplitude as with real modes, but also phase, which results in the appearance of travelling waves; this contrasts with real modes which are better described by standing waves.

DESCRIPTION OF BOUNDARY CONDITIONS AT THE HUB

The coupling of the wheel and the vehicle substructures can be achieved either with spatial models, modal models or response models. For this investigation the latter method is employed to calculate the coupled oscillation response of the wheel and vehicle. The response method for coupling is based on the conditions of compatibility and equilibrium at the interface degrees of freedom which are common to the two substructures. The receptance version of the method used to calculate the coupled FRF matrix $[H_C]$ from the two uncoupled FRF matrices $[H_A]$ and $[H_B]$ is given by Eq. 3:

Eq. 3

$$[H_C]^{-1} = [H_A]^{-1} \oplus [H_B]^{-1}$$

The receptance, or frequency response defined as displacement per unit force, of the wheel substructure is calculated

FIGURE 1 FE model of the vehicle with about two million nodes for the chassis (© Daimler)

FIGURE 2 Test setup with 3-D scanning laser Doppler vibrometer (left) and wheel test specimen on the roller drum test rig (© Continental)

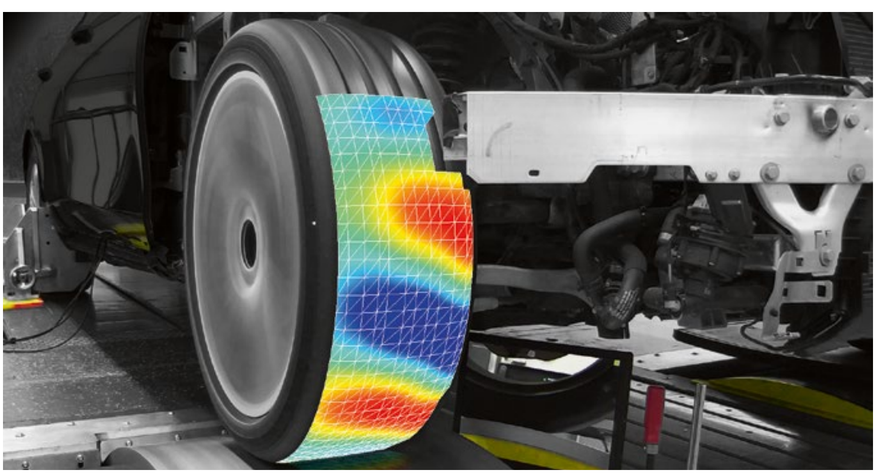


FIGURE 3 Measured operating deflection shape of a tread segment at 90 km/h (© Continental)

herein with the modal model presented above. One advantage of the response method for this application is that due to the small number of degrees of freedom needed at the coupling point between the wheel and hub, the matrices to be inverted in Eq. 3 are very small.

The FE model of the vehicle, **FIGURE 1**, used in this project, includes a detailed chassis (about two million nodes) and the powertrain as well as the complete body structure (about five million nodes). The 36 Frequency Response Functions (FRF) which are used for the tire-to-vehicle connection are obtained by computing the wheel hub's receptance matrix F/X in all six directions. Further frequency response functions can be computed from the wheel center to chassis or body points.

TEST SETUP

For the test rig at Continental, a 3-D scanning laser Doppler vibrometer [3, 4] from Polytec is used. The front right wheel is positioned on the roller drum, **FIGURE 2**. The surface of the drum is machined with small non-uniformities and applies a known excitation without loss of contact. Therefore, at every speed a defined kinematic excitation is applied to the tire at a finite set of frequencies which are proportional to the drum rotational speed.

The wheel was driven by the drum at constant speeds between 33 and 99 km/h (in steps of 3 km/h). At each speed step the wheel was scanned from nine different viewing angles, four of which were different orientations of a mirror placed behind the wheel.

In order to obtain the three-dimensional operational response vectors a test setup with the vibrometer is applied that makes use of three laser sensors incident from different viewing angles. The sensors of the vibrometer detect the Doppler shift proportional to the instantaneous velocity at the tire surface in the direction of the laser beam. A coordinate transformation reveals the x-, y- and z-components of the vibration vector. By synchronously scanning the three beams the vibration field can be measured, including the phases.

TEST RESULTS

The wheel response, and therefore the interior noise of the vehicle, is largely driven by the characteristics of a small number of tire modes. These modes have shapes which lead to a large resultant force at the axle and are normally the first in a family of mode shapes, that means, first lateral model, first vertical mode, first cavity mode, etc.

During the scanning of the rolling wheel, the sound pressure levels at four positions in the cabin were recorded. Peaks in the spectra of these signals can be easily identified below 300 Hz, and the corresponding measured Operating Deflection Shapes (ODSs) of the wheel at the same frequencies can be analyzed, **FIGURE 3**. For many of these operation points the ODSs are driven by the tire modes that we would expect.

With the models presented, the ODSs of the rolling tire-wheel system can be calculated at the frequencies identified by measurement. For this purpose, the forces at the interface degrees of freedom between the wheel and axle are first calculated using the response coupling method. These forces are then applied to the modal tire-wheel model together with the kinematic excitation at the contact patch. This enables the calculated and measured vibration fields on the surface of the tire and wheel to be compared.

Setting the same scaling for both measured and calculated displacements, the ODSs of several important tire-wheel resonances are shown in **TABLE 1**. The resultant instantaneous amplitude of velocity is represented as a color. Here, it is fascinating to see that the ODSs, assessed by calculation, can actually be revealed by measurement techniques at rolling speeds of up to 99 km/h. The crucial aspect of rotation [5, 6] is mastered both in experiment and calculation.

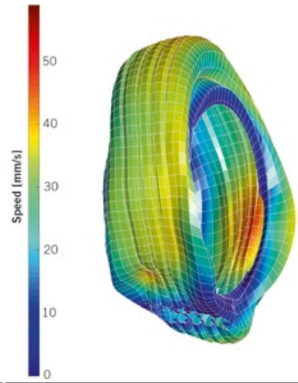
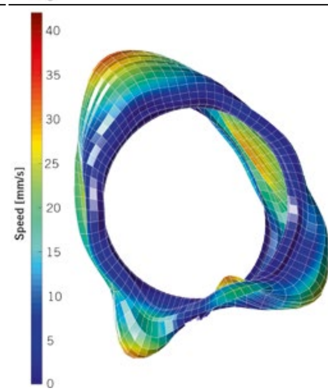
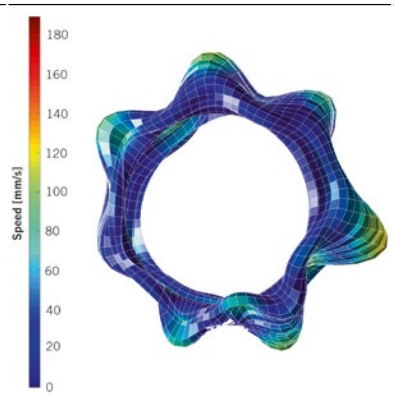
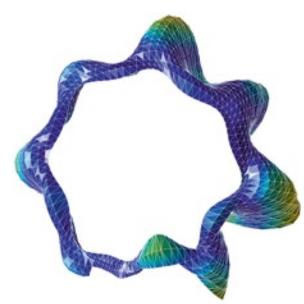
Description	Calculation	Measurement
ODSs at 97 Hz and 84 km/h; first vertical mode of the tire, visible in response		
ODSs at 115 Hz and 48 km/h; second radial mode of the tire, visible in response		
ODSs at 216 Hz and 90 km/h; this is the frequency of the cavity mode		

TABLE 1 Comparison of calculation and measurement for selected frequencies (© Continental)

SUMMARY AND OUTLOOK

This paper presents a detailed investigation on tire rolling noise and vibration using advanced tire and vehicle models as well as innovative non-contact laser vibrometry. It was shown that a very good agreement can be achieved between calculation and measurement, despite challenging conditions in both fields.

The tire model used in this study was set up based only on design specifications, with no data from the physical tire being utilized for the setup of the model. In the development process this would also be the case, given that there is normally a demand for numerical predictions before the tire is produced. Due to the large amount of high quality data collected by the vibrometer, the models could be further improved by employing a suitable updating procedure. This could lead to an even better understanding of the mechanics of the rolling tire and wheel, and would therefore be an interesting topic for further research.

Based on measurements of a larger set of tire specimens, using the same test setup described herein, the quality of the simulation method can be validated. This is the foundation for the application of the numerical method in the early development stage, for the optimization of both the tire and the vehicle for increased NVH performance.

THANKS

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